Coding Theory
and
Digital Data Transmission
Lesson VI

Joel Isaacson
Bar-Ilan Computer Science

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1 Bandpass Modulation

Communication over a bandpass communication channel, such as a radio or optical link, usually requires that the signal of interest is modulated to the bandpass frequency. Modulation is the act of shifting the signal in the frequency domain, so that the signal is only in the specific bandpass region. There are two types of **continuous-wave** (CW) modulation schemes:

1. Amplitude Modulation (AM)
2. Phase or frequency modulation (FM).

In CW modulation schemes a **carrier signal,** $A_c \cos(\omega_c t)$,is used to "modulate" the original signal. In general a modulated carrier signal can be represented mathematically as

$$x_c(t) = A(t)\cos[\omega_c t + \phi(t)] \quad \omega_c = 2\pi f_c. \tag{1}$$

In eqn. 1 $f_c = \frac{\omega_c}{2\pi}$ is known as the **carrier frequency**. $A(t)$ and $\phi(t)$ are the **instantaneous amplitude** and **phase angle** of the carrier respectively. If $\phi(t) = C_\phi$ and $A(t) \neq C_A$ then the modulation technique is called **Amplitude Modulation** (AM). If the Amplitude is constant, $A(t) = C_A$ and the phase is a function of time, $\phi(t) \neq C_\phi$, we have **Phase or Frequency Modulation** (PM or FM).

2 Amplitude Modulation

2.1 Double side band

In this scheme we just multiply the carrier wave by the signal which we want to transmit,

$$x_{DSB}(t) = m(t)\cos(\omega_c t), \tag{2}$$

where the message to be transmitted is just $m(t)$. We can easily find the Fourier transform of this signal, since the cosine is just $\frac{1}{2}(e^{i\omega_c t} + e^{-i\omega_c t})$, the multiplication translates the signal $m(t)$ in frequency $\pm\omega_c$. Thus the Fourier transform of $x_c(t)$ is:

$$X_{DSB}(\omega) = \frac{1}{2}M(\omega - \omega_c) + \frac{1}{2}M(\omega + \omega_c). \tag{3}$$

This is shown in fig. 1 Notice that the signal in positive frequencies is replicated twice. This is not an artifact of the Fourier analysis. There is a duplication of the information of the original signal, that can be

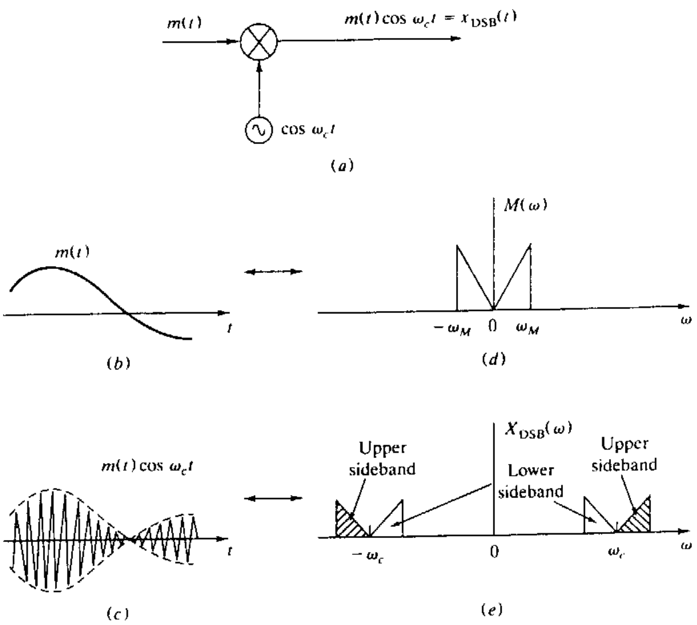

Figure 1: AM - Double Sideband Modulation

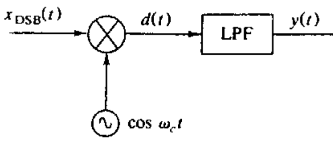

Figure 2: DSB Demodulation

recovered from either the upper or lower sideband. We are expending double the transmission power in order to transmit the signal twice.

We can **de-modulate** (recover the original signal) by multiplying the modulated signal by a local carrier signal, $\cos(\omega_c t)$, fig. 2 Note that the phase and frequency of the carrier signal must be synchronized with the remote carrier frequency if the demodulation is distortion free.

2.2 Ordinary Amplitude Modulation

In this AM technique we add a fixed value to the input signal,

$$m^{'}(t) = A + m(t), \tag{4}$$

so that it is a positive function in time. This has the effect of added a large un-modulated carrier signal to the transmitted signal

$$x_{AM}(t) = m(t)\cos(\omega_c t) + A\cos(\omega_c t). \tag{5}$$

The spectrum of $x_{AM}(t)$ is given by

$$X_{AM}(\omega) = \frac{1}{2}M(\omega - \omega_c) + \frac{1}{2}M(\omega + \omega_c) + A\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)], \tag{6}$$

as shown in fig. 3 Notice that in addition to the DSB signal we have a rather large delta function at the

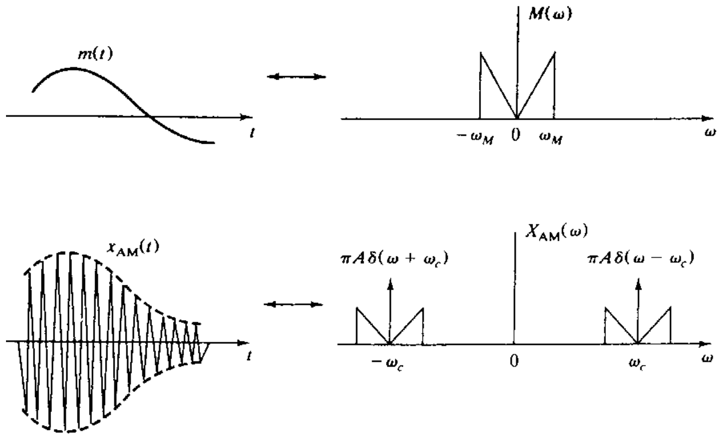

Figure 3: Ordinary Amplitude Modulation

carrier frequency that consumes much transmission power. Also we have to be careful that the original signal, $m(t)$ does not get to large so as to cause $m^{'}(t)$ to go negative, this is called **over-modulation**.

We can rather simply recover the signal by looking at the upper (or lower) envelop. The reason for this is that the carrier wave comes from the sender and the there is no need to create a synchronized carrier wave at the receiver to demodulate the signal.

2.3 Single Side Band Modulation

Both bandwidth and power are wasted in DSB and ordinary AM modulation. Since either the upper or lower sideband contain the original signal we really only have to send one of these sidebands. Fig. 5 show conceptually how this can be done using an ideal (non-practical) bandpass filter. If we can regenerate a carrier wave we can simply recreate the original signal $m(t)$ as shown in fig. 6.

2.4 Frequency Translation

We frequently have need to translate the frequency of signals. As we have seen this can be accomplished easily by multiplying by a mixing signal $2\cos(\omega_2 t)$, as show in fig. 7 The operation is done in our standard AM superhetrodyne receivers where all signals are translated to 455 kHz before amplification. Thus if we want to to tune into 600 kHz we mix the signal with a mixing signal of 1055 kHz. This will down-translate the original 600 kHz band into the 455 kHz band ($\omega_1 = 600$, $\omega_2 = 1055$, $\pm\omega_1 \mp \omega_2 = \mp 455$). Also notice that the frequency 1510 kHz will also be down-translated into 455 kHz ($\omega_1 = 1510$, $\omega_2 = 1055$, $\pm\omega_1 \mp \omega_2 = \pm 455$). This means that the signal modulated at 1510 kHz will interfere with the signal modulated at 600 kHz. This signal must be filtered out before the mixing.

2.5 Frequency-Division Multiplexing

If we have a shared media we will want to allocate “bands” for each different signal. This might be radio, a microwave link or an optical fiber. We normally split the bandwidth up into ranges for each signal and simultaneously send the signals over the media. If each signal doesn’t use more bandwidth than allocated we can recover all the signal without mutual interference. This is shown in fig. 8

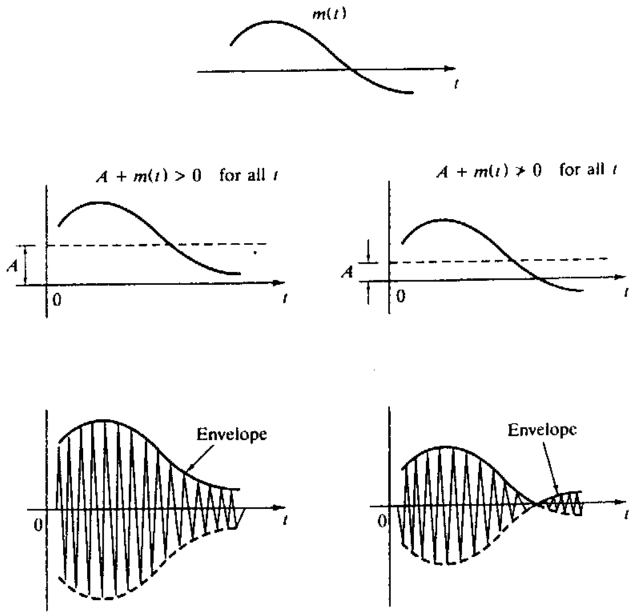

Figure 4: AM signal and its envelope

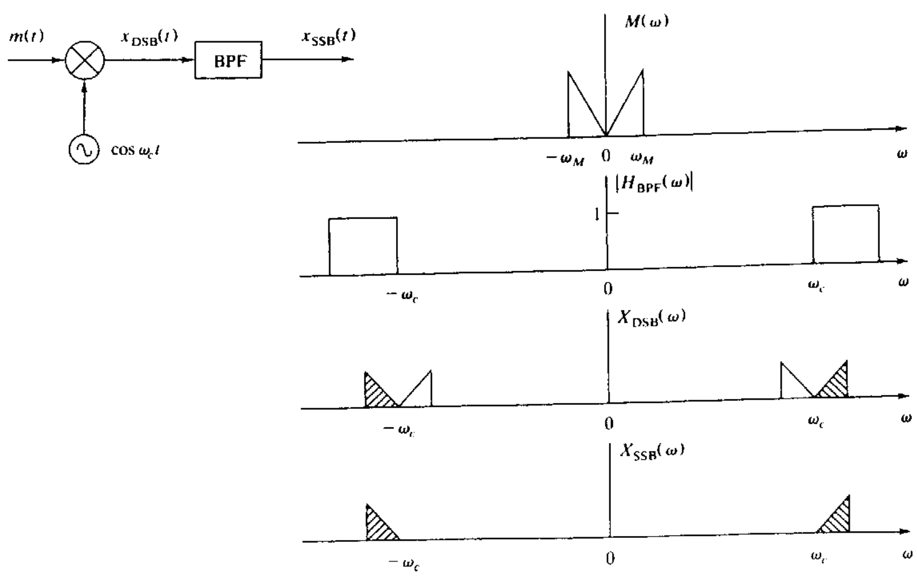

Figure 5: Single Side Band using a bandpass filter

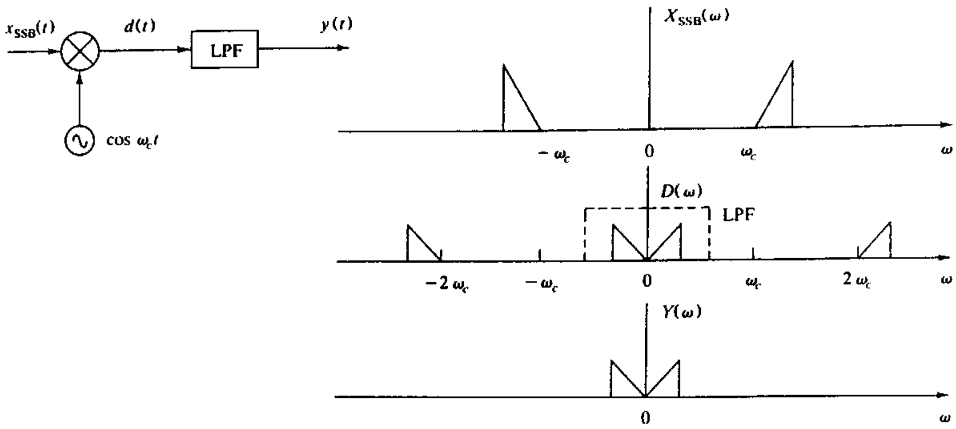

Figure 6: Demodulation of the SSB signal

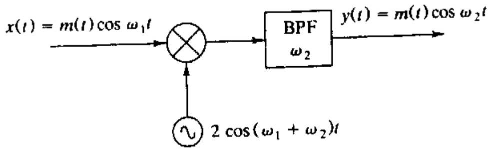

Figure 7: Frequency mixer

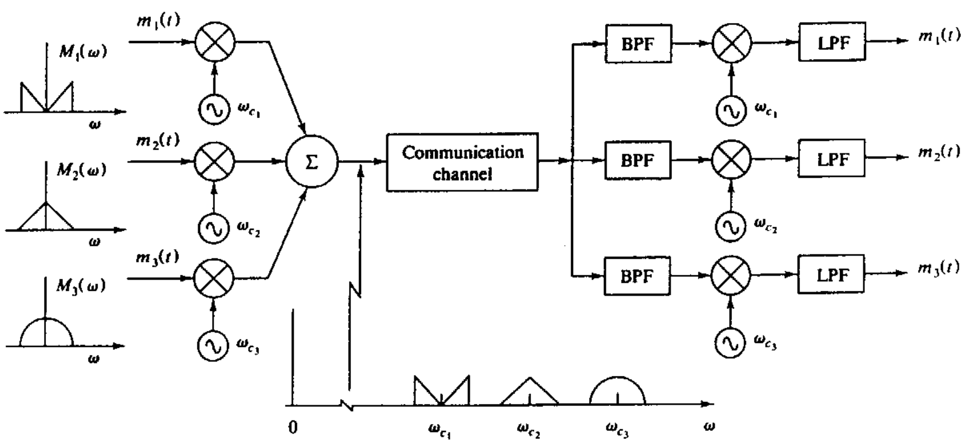

Figure 8: Frequency-Division Multiplexing

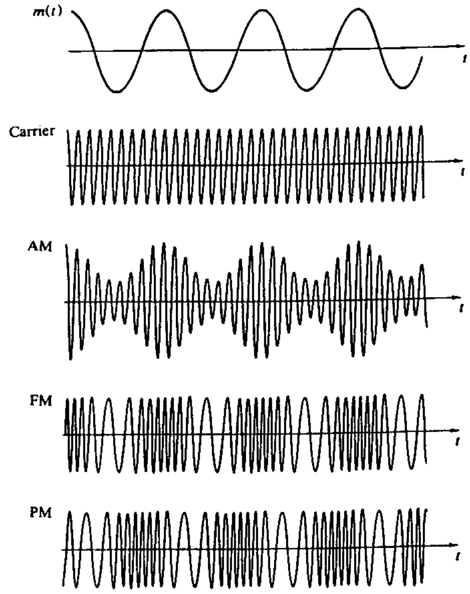

Figure 9: AM, FM and PM waveforms

3 Angle Modulation

3.1 Phase and Frequency Modulation

In this scheme the amplitude of the signal is constant but the frequency is modulated

$$x_c = A\cos\theta(t) \tag{7}$$

where

$$\theta(t) = \omega_t t + \phi(t), \tag{8}$$

where $\phi(t)$ is called the **instantaneous phase deviation**. We can define the **instantaneous frequency frequency** of $x_c(t)$ as

$$\omega_i = \frac{d\theta(t)}{dt} = \omega_t + \frac{d\phi(t)}{dt}. \tag{9}$$

In phase modulation we have the instantaneous phase deviation of the carrier proportional to the message signal

$$\phi(t) = k_p m(t). \tag{10}$$

In frequency modulation we have the instantaneous frequency deviation proportional to the signal $m(t)$,

$$\frac{d\phi(t)}{dt} = k_f m(t) \tag{11}$$

or

$$\phi(t) = k_f \int_0^t m(\lambda)d\lambda + \phi_0(t) \tag{12}$$

Fig. 9 shows an example of the three different modulations.