Coding Theory
and
Digital Data Transmission
Lesson VIII

Joel Isaacson
Bar-Ilan Computer Science

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1 Channel capacity

We can use the inverse of the Nyquist theorem to know what bandwidth is needed to sustain a particular **baud** (symbol or sample per second) rate. If we want to transmit a signal at B baud we need a bandwidth of $B/2$. The maximum information content of a signal is just

$$I = Bn \tag{1}$$

where n is the number of bits per sample.

1.1 Example of optimal encoding

Let us give an example of the optimal encoding using a simple amplitude modulation scheme. We want to create a continuous function that will enable us to encode a discrete number of "code" symbols over time. In order to prevent inter-symbol interference (fig. 1) we will need to find a continuous function, $S(t)$, that will have a non zero value (1) at 0 and zero values at all $t = Tn$ while $n \neq 0$. In other words $S(Tn) = \delta_{n,0}$. By the fundamental Theorem of Algebra knowledge of the zeros of an algebraic function uniquely gives the function (fig. 2) thus,

$$S(t) = \prod_{n=1}^{\infty} \left(1 - \frac{(t/T)^2}{n^2}\right). \tag{2}$$

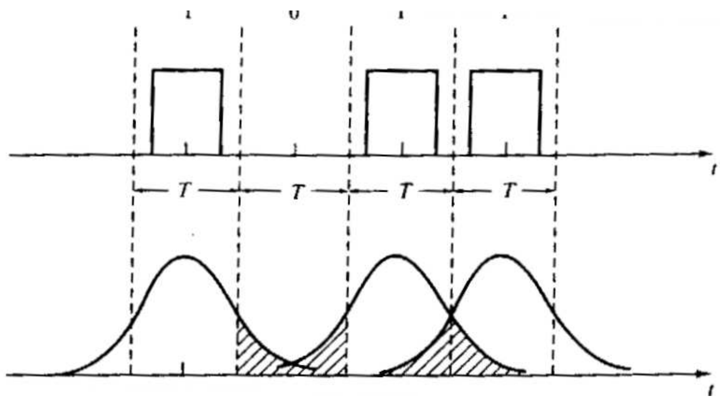

Figure 1: Inter Symbol Interference

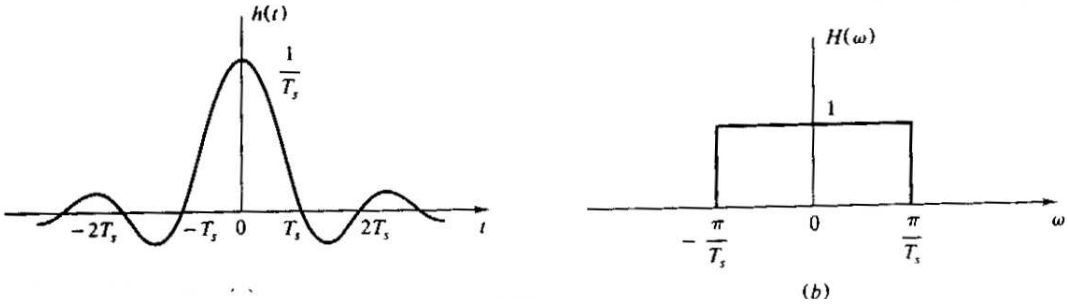

Figure 2: Single bit encoding function

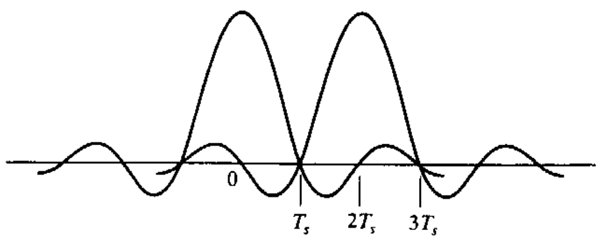

Figure 3: 0-1-0-1-0 encoded

By the same logic we can write a infinite product for the $\sin$ function,

$$\sin(t) = x \prod_{n=1}^{\infty} \left(1 - \frac{t^2}{n^2\pi^2}\right). \tag{3}$$

Thus the relationship between $S(t)$ and $\sin(t)$ is:

$$S(t) = \frac{T}{\pi t} \sin(\frac{\pi t}{T}) = \mathrm{sinc}(\frac{t}{T}). \tag{4}$$

As we know from previous lectures the bandwidth of this function is $B/2$, where $B = 1/T$. Thus the minimum bandwidth needed to transmit $1/T$ symbols per second is at least $B/2$, which is the result that we would have gotten by us of Nyquist's theorem. Fig 3 shows the data $...0-1-0-1-0...$ encoded optimally with the minimum Nyquist bandwidth.

1.2 The Shannon-Hartley theorem

We of course can try to transmit an unlimited number of quantized levels per symbol but the intrinsic noise level of the channel will limit the error free reception of the symbols at the receiver. The maximal information capacity as a function of the signal to noise ratio S/N was given in a famous result by Shannon-Hartley

$$I = B \log_2(1 + S/N). \tag{5}$$

Thus for a telephone channel of 3K Khz with a signal to noise ratio of 30dB ($10 \log_{10} \frac{S}{N} = 30$ or $\log_2(1 + \frac{S}{N}) \simeq \log_{10}(1 + \frac{S}{N})/\log_{10} 2 \simeq 3/.30103... \simeq 10$), the total channel capacity is less than 30kbps.

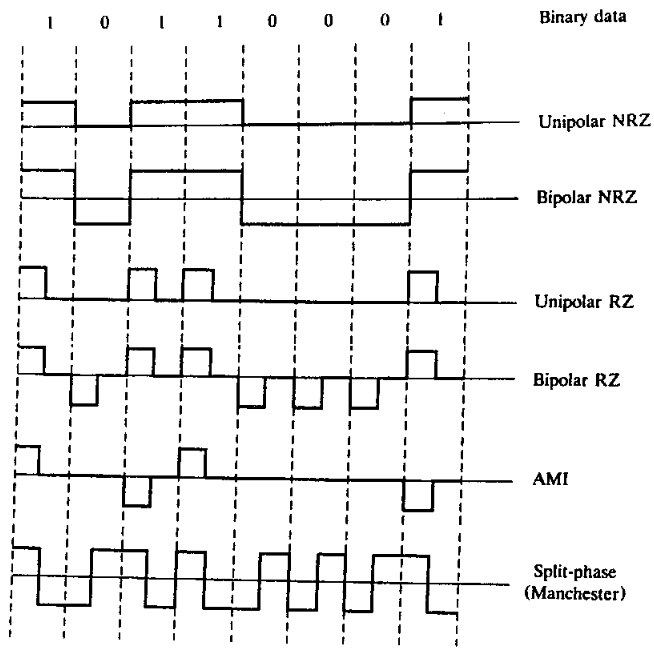

Figure 4: Baseband binary signaling formats

2 Line Coding

The simplest data transmission is via a data bus. This is the usual means of transmission of the data within a computer and for short distances such a parallel printer interfaces. For large distances parallel data transmission over different channels is not practical due to the phase (time) differences between the different channels that become more severe as the distances become longer.

For digital signals that are transmitted on one channel in **baseband** (unmodulated) mode we have many different formats used fig. 4.

In order to send data over a serial channel with no shared clock signal. A frequent technique to synchronize clocks on the two ends of a communication channel is by a **phase lock loop.** A phase-locked loop (PLL) is an electronic circuit with a voltage- or current-driven oscillator that is constantly adjusted to match in phase (and thus lock on) the frequency of an input signal. The phase of the receiving clock "locks" on to the transmitted signal to correct the drift in the clock.

Non-return to zero encoding is commonly used in slow speed communications interfaces for both synchronous and asynchronous transmission. Using NRZ, a logic 1 bit is sent as a high value and a logic 0 bit is sent as a low value (the line driver chip used to connect the cable may subsequently invert these signals).

A problem arises when using NRZ to encode a synchronous link which may have long runs of consecutive bits with the same value. The figure below illustrates the problem that would arise if NRZ encoding were used with a PLL recovered clock signal. In Ethernet for example, there is no control over the number of 1's or 0's which may sent consecutively. There could potentially be thousands of 1's or 0's in sequence. If the encoded data contains long 'runs' of logic 1's or 0's, this does not result in any bit transitions. The lack of transitions (fig. 5)prevents the receiver PLL from reliably regenerating the clock making it impossible to detect the boundaries of the received bits at the receiver.

802.3 Ethernet uses Manchester Phase Encoding (MPE). A data bit '1' from the level-encoded signal (i.e. that from the digital circuitry in the host machine sending data) is represented by a full cycle of the inverted signal from the master clock which matches with the '0' to '1' rise of the phase-encoded signal (linked to the phase of the carrier signal which goes out on the wire). i.e. -V in the first half of the signal and +V in the second half.

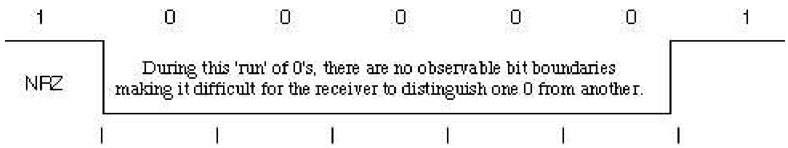

Figure 5: A run of zeros in simple NRZ coding

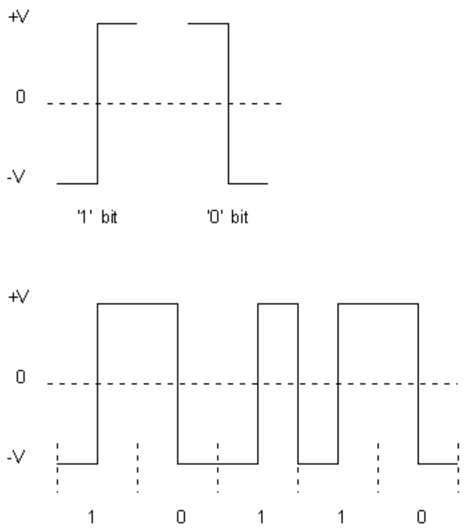

Figure 6: Manchester Phase Encoding (MPE)

The data bit '0' from the level-encoded signal is represented by a full normal cycle of the master clock which gives the '1' to '0' fall of the phase-encoded signal. i.e. +V in the first half of the signal and -V in the second half.

The fig. 6 shows graphically how MPE operates. The example at the bottom of the diagram indicates how the digital bit stream 10110 is encoded.

A transition in the middle of each bit makes it possible to synchronize the sender and receiver. At any instant the ether can be in one of three states: transmitting a 0 bit (-0.85v), transmitting a 1 bit (0.85v) or idle (0 volts). Having a normal clock signal as well as an inverted clock signal leads to regular transitions which means that synchronisation of clocks is easily achieved even if there are a series of '0's or '1's. This results in highly reliable data transmission. The master clock speed for Manchester encoding always matches the data speed and this determines the carrier signal frequency, so for 10Mbps Ethernet the carrier is 10MHz.

More sophisticated codes (fig. 1)are used in modern baseband physical layers. Take Gigabyte Ethernet 1000 BASE-X which uses 8B/10B for example. Since a signal that contains no data transitions has a DC component that causes problems with line isolation (capacitance or via a transformer) and clock synchronization we must guarantee a certain number of phase transitions and balanced (no net DC) operation.

3 Modulated Signaling (Bandpass Digital Transmission)

Modulated formats are seen in fig. 7. Any modulated bandpass signal may be expressed in the quadrature-carrier form

$$x_{c(t)} = A_c(x_i(t)\cos(\omega_c t + \theta) - x_q(t)\sin(\omega_c t + \theta)) \tag{6}$$

Value	Data Byte	Manchester Encoding 1B/2B	8B/10B	5B/6B	3B/4B
00	0000 0000	01010101 01010101	011000 1011	011000	0100
01	0000 0001	01010101 01010110	100010 1011	100010	1001
02	0000 0010	01010101 01011001	010010 1011	010010	0101
04	0000 0100	01010101 01100101	001010 1011	001010	0010
07	0000 0111	01010101 01101010	000111 0100	000111	0001
08	0000 1000	01010101 10010101	000110 1011	000110	-
0F	0000 1111	01010101 10101010	101000 1011	101000	-
F0	1111 0000	10101010 01010101	100100 1110	-	-
FF	1111 1111	10101010 10101010	010100 1110	-	-

Table 1: Several Data Encoding

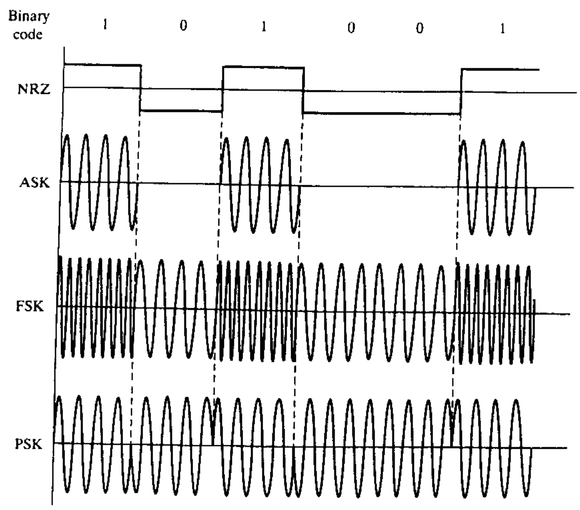

Figure 7: Modulated binary signaling formats

where the frequency f_c, amplitude A, and phase θ are constant. If we define the power spectrum of $x_i(t)$ to be $G_i(f)$ and $x_q(t)$ to be $G_q(f)$ we can define the low-pass spectrum to be

$$G_{lp}(f) = G_i(f) + G_q(f). \tag{7}$$

We can show that the spectrum for the signal is then

$$G_c(f) = \frac{A_c}{4}\left(G_{lp}(f - f_c) + G_{lp}(f + f_c)\right) \tag{8}$$

3.1 ASK - Amplitude shift keying.

In ASK we turn the carrier on and off using a process called **on-off keying**. Here the function $x_i(t)$ is just a sum of square waves

$$x_i(t) = \sum_k a_k p_D(t - kD) \tag{9}$$

where

$$p_D(t) = \begin{matrix} 1 & |D| < \tau/2 \\ 0 & |D| \geq \tau/2 \end{matrix} . \tag{10}$$

In the case of on-off keying a_k is either 0 or 1. We normally have to analyse this function for random a_k to get the average spectrum or use a measured distribution of $a_{k.}$. In order to understand the "worst" case we can take

$$a_k = \begin{matrix} 0 & k \text{ even} \\ 1 & k \text{ odd} \end{matrix} . \tag{11}$$

This is the case with the most variance in the signal $x_i(t)$ and it should give us an upper bound for the bandwidth needed to transmit this signal. The function $x_i(t)$ in this case is just a pulse train with periodicity $2D$ each pulse having a width of D. Since this signal has periodicity, $2D$, the Fourier transform has a discrete sum of dirac delta (impulses) nature, or is just a Fourier series. The Fourier series for this pulse train was solved in lecture 2. Quoting from lecture 2.

The Fourier coefficients, c_n, are thus,

$$\begin{aligned} c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} r(t)e^{-i2\pi n f_0 t}dt &= \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} Ae^{-i2\pi n f_0 t}dt \\ &= \frac{A}{-i2\pi n f_0 T_0}(e^{-i2\pi n f_0 \tau} - e^{+i2\pi n f_0 \tau}) \\ &= \frac{A\tau}{T_0}\,\mathrm{sinc}\,(n f_0 \tau). \end{aligned} \tag{12}$$

Here $\tau = D$ and $T_0 = 2D$ and the bandwidth of the power spectrum (c_n^2) is approximately $\frac{1}{D}$. In the case of a random signal a_k the major change is that the spectrum is continuous but the result is similar. This means that in order to send $r = 1/D$ symbols per second we take up a bandwidth of r centered around f_0. Of course we also are transmitting some wicked side-lobes (spillover) to the spectrum that might case interference with adjacent FDM (frequency domain multiplexing) channels. Note that the power spectrum is the signal spectrum squared which lowers the side-lobes. We also can add some filtering that will decrease the amplitude of the side-lodes.

This is in line with what we would expect from Nyquist's theorem except for the factor of 2. We even can transmit a second signal, $x_q(t)$ that is 180^o out of sync with $x_i(t)$ and approach the Nyquist limit. This is called **quadrature-carrier** AM (QAM) and it achieves twice that rate of a simple ASK channel

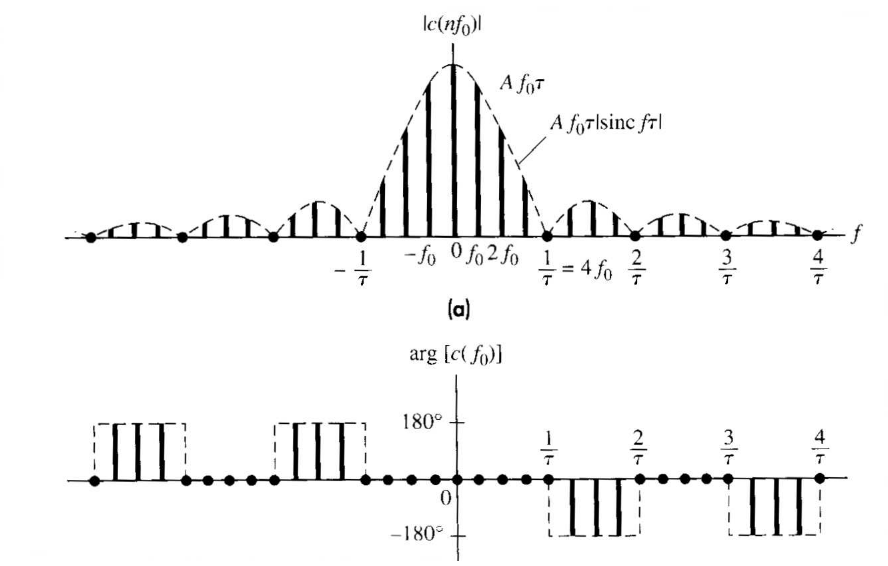

Figure 8: Spectrum of rectangular pulse with $f_0\tau = 1/4$.(a) Amplitude (b) Phase