Coding Theory
and
Digital Data Transmission
Lesson IX

Joel Isaacson
Bar-Ilan Computer Science

22nd May 2005

1 Modulated Signaling (Bandpass Digital Transmission)

Modulated formats are seen in fig. 1. Any modulated bandpass signal may be expressed in the quadrature-carrier form

$$x_{c(t)} = A_c(x_i(t)\cos(\omega_c t + \theta) - x_q(t)\sin(\omega_c t + \theta)) \tag{1}$$

where the frequency f_c, amplitude A, and phase θ are constant. If we define the power spectrum of $x_i(t)$ to be $G_i(f)$ and $x_q(t)$ to be $G_q(f)$ we can define the low-pass spectrum to be

$$G_{lp}(f) = G_i(f) + G_q(f). \tag{2}$$

We can show that the spectrum for the signal is then

$$G_c(f) = \frac{A_c}{4}\left(G_{lp}(f - f_c) + G_{lp}(f + f_c)\right) \tag{3}$$

1.1 ASK - Amplitude shift keying.

In ASK we turn the carrier on and off using a process called **on-off keying**. Here the function $x_i(t)$ is just a sum of square waves

$$x_i(t) = \sum_k a_k p_D(t - kD) \tag{4}$$

where

$$p_D(t) = \begin{matrix} 1 & |D| < \tau/2 \\ 0 & |D| \geq \tau/2 \end{matrix}\ . \tag{5}$$

In the case of on-off keying a_k is either 0 or 1. We normally have to analyse this function for random a_k to get the average spectrum or use a measured distribution of a_k.. In order to understand the “worst” case we can take

$$a_k = \begin{matrix} 0 & k \text{ even} \\ 1 & k \text{ odd} \end{matrix}\ . \tag{6}$$

This is the case with the most variance in the signal $x_i(t)$ and it should give us an upper bound for the bandwidth needed to transmit this signal. The function $x_i(t)$ in this case is just a pulse train with periodicity $2D$ each pulse having a width of D. Since this signal has periodicity, $2D$, the Fourier transform has a discrete sum of dirac delta (impulses) nature, or is just a Fourier series. The Fourier series for this pulse train was solved in lecture 2. Quoting from lecture 2.

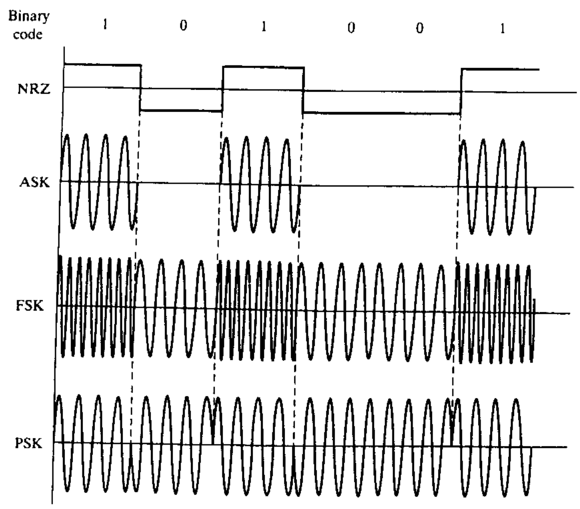

Figure 1: Modulated binary signaling formats

The Fourier coefficients, c_n, are thus,

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} r(t)e^{-i2\pi n f_0 t}dt = \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} Ae^{-i2\pi n f_0 t}dt$$
$$= \frac{A}{-i2\pi n f_0 T_0}(e^{-i2\pi n f_0 \tau} - e^{+i2\pi n f_0 \tau})$$
$$= \frac{A\tau}{T_0}\,\mathrm{sinc}\,(n f_0 \tau). \tag{7}$$

Here $\tau = D$ and $T_0 = 2D$ and the bandwidth of the power spectrum (c_n^2) is approximately $\frac{1}{D}$. In the case of a random signal a_k the major change is that the spectrum is continuous but the result is similar. This means that in order to send $r = 1/D$ symbols per second we take up a bandwidth of r centered around f_0. Of course we also are transmitting some wicked side-lobes (spillover) to the spectrum that might case interference with adjacent FDM (frequency domain multiplexing) channels. Note that the power spectrum is the signal spectrum squared which lowers the side-lobes. We also can add some filtering that will decrease the amplitude of the side-lodes.

This is in line with what we would expect from Nyquist's theorem except for the factor of 2. We even can transmit a second signal, $x_q(t)$ that is 180^o out of sync with $x_i(t)$ and approach the Nyquist limit. This is called **quadrature-carrier** AM (QAM) and it achieves twice that rate of a simple ASK channel.

1.2 M-ary QAM Systems

We can combine the amplitude modulation technique with the phase shift keying to modulate a signal in two dimensions, amplitude and phase. For example we can pick two levels of amplitude modulation and four phase shift values to transmit 8 different values (three bits) per baud. In this case we have table 1. Notice that this scheme can be represented by eq. 1, with $x_i(t)$ and $x_q(t)$ as shown in the last column.

Let us suppose that want to transmit the bit stream:

$$001010100011101000011110$$

We first break up this stream into 3-bit symbols:

$$001 - 010 - 100 - 011 - 101 - 000 - 011 - 110$$

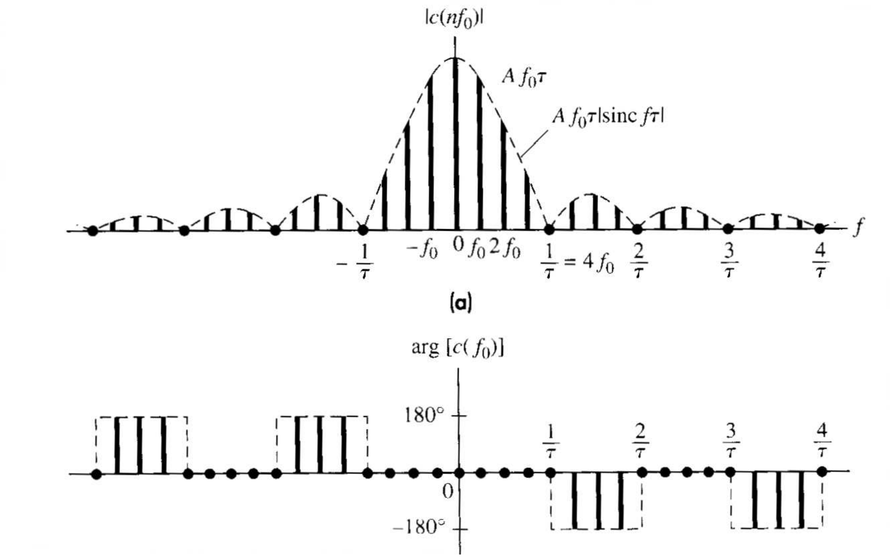

Figure 2: Spectrum of rectangular pulse with $f_0\tau = 1/4$.(a) Amplitude (b) Phase

Bit Value	Amplitude	Phase Shift	x_i, x_q
000	1	0	$1, 0$
001	2	0	$1, 0$
010	1	1/4	$\sqrt{2}, -\sqrt{2}$
011	2	1/4	$\sqrt{2}, -\sqrt{2}$
100	1	1/2	$0, 1$
101	2	1/2	$0, 1$
110	1	3/4	$\sqrt{2}, -\sqrt{2}$
111	2	3/4	$\sqrt{2}, -\sqrt{2}$

Table 1: 2,4-ary QAM

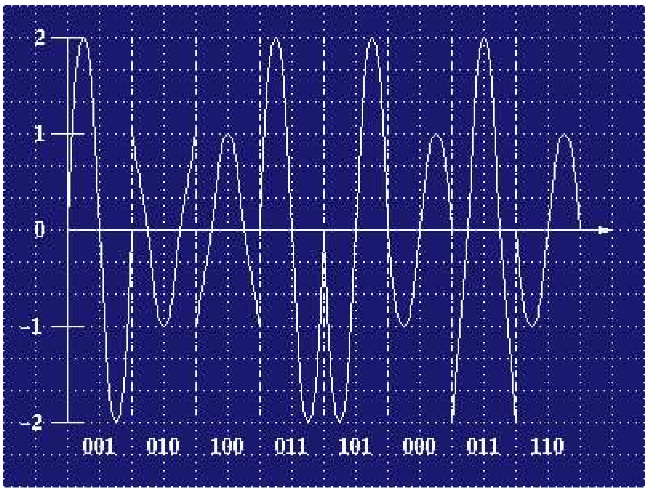

Figure 3: The bit stream encoded with three bit triads.

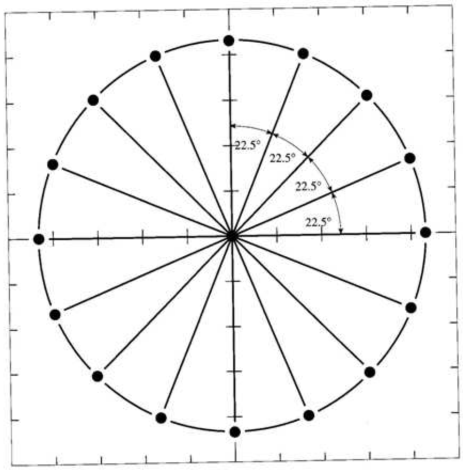

Figure 4: 16-ary PSK

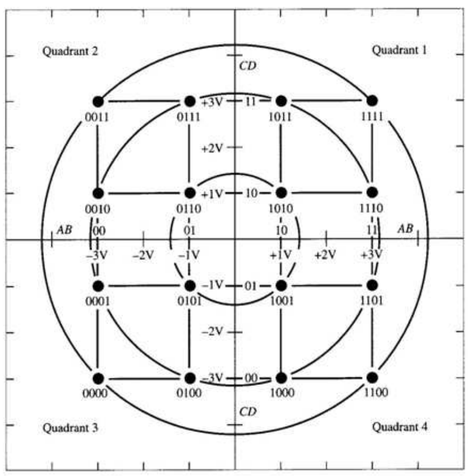

Figure 5: 16QAM

We then can encode the stream as shown in figure 3. The only thing different about this figure than the previously explained scheme is that the phase is not absolute, as shown in the table, but rather relative to the previous phase.

1.3 Signal Constellation Diagrams

We can plot out the the signal space in two dimensions, amplitude and phase. For example if we take a purely phase shift keying (PSK) modulation technique, we have constant amplitude. The different signal states is shown in figure 4 For a 16-ary QAM system the signal constellation diagram is shown in fig. 5 The signal constellation diagrams for different QAM systems is shown in figure. 6

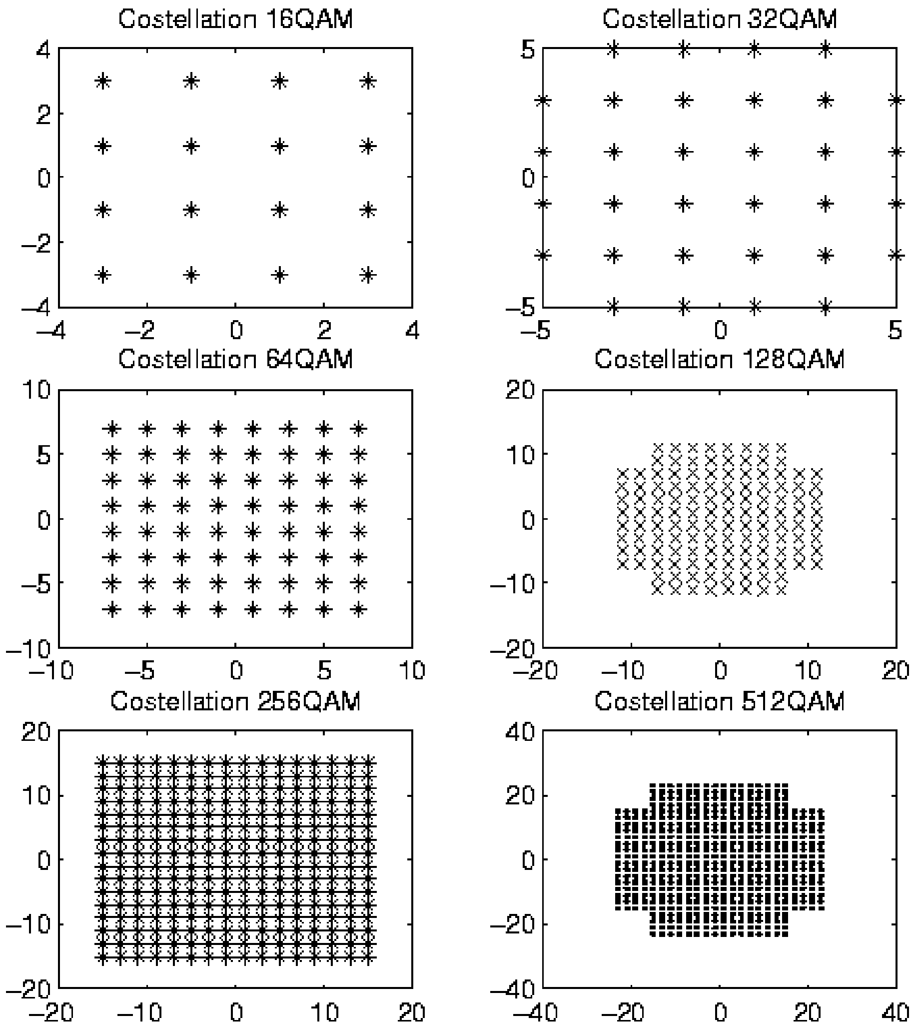

Figure 6: QAM Constellation diagrams.