Photorealistic Rendering
Lesson V

Joel Isaacson Bar-Ilan
Computer Science

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1 Light and Matter

1.1 Light

In image synthesis we are interested in the simulation of light. Light has a famous dual nature, it sometimes acts as a particle and in other times it acts as a though it is a wave.

Light is a form of electromagnetic radiation that can be described by four famous equations known as *Maxwell's equations*. Due to this wave nature, light can be decomposed into components having well defined wavelengths λ. If we have source of light I we can decompose the light radiation into a spectrum of different wavelengths:

$$I = \int_0^\infty I(\lambda) d\lambda \tag{1}$$

Light also can be polarized along the two planes perpendicular to the direction of propagation. The wave nature of light can be demonstrated by diffraction of light waves by narrow slits and interference effects.

The particle nature of light can be demonstrated by the well know photoelectric effect. This effect cause Einstein to postulate the existence of quantized individual packets of light called photons. The energy, E of an individual photon is inversely related to the wavelength of the light:

$$E = hc/\lambda \tag{2}$$

Here c is the speed of light and h is *Planck's* constant ($6.62620 \times 10^{-32} J \cdot s$).

The contradiction of the particle versus wave behavior of light is addressed by quantum theory. We will limit our treatment of light to the particle nature of light. The theory of light that explains the propagation of light particles is called *geometrical optics*. This theory can describes most effects of interest. There are two effects that can't be easily handled by geometrical optics, *interference* and *diffraction*. Interference accounts for the colors seen on thin films, This includes peacock feathers, oil slicks and soap bubbles. Diffraction is responsible for some of the fuzziness of shadows and for light bleeding around the edge of objects.

1.2 Reflection and transmission

1.2.1 Reflection

Reflection is the process in which light of a specific wavelength incident on a material is wholly or partially propagated outwards from that material without changing wavelength. Most models of reflection are classified into a few different categories as shown in Fig 1.

Specular reflection propagates light without scattering. This is the type of reflection off a mirror.

Diffuse reflection send light in all directions with equal energy.

Mixed reflection is a combination of the two types described above.

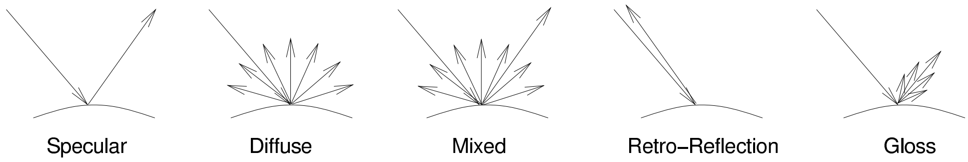

Figure 1: Different categories of reflection

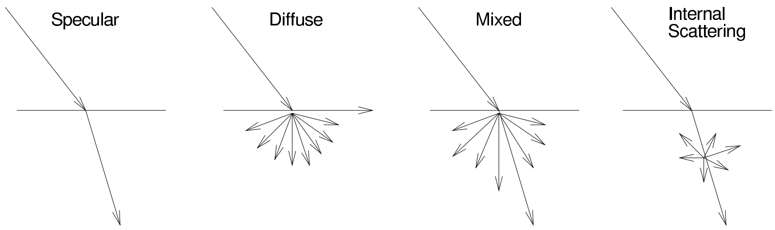

Figure 2: Different categories of transmission

Retro-reflection is when a significant amount of the incident energy is back reflected at an angle that is close to the angle of incidence.

Gloss reflection is reflection that is preferentially but not exclusively in the specular direction.

1.2.2 Transmission

Transmission (or refraction) is the process in which light incidence on a material passes through the material without change in wavelength. Here to we too have a number of different categories as shown in Fig. 2:

Specular transmission propagates light into the new material without scattering at a new angle called the angle of refraction.

Diffuse transmission has the light propagating into the material and scattering equally in all directions.

Mixed transmission is a combination of specular and diffuse.

Scattering within the material

When light moves from a less dense to a more dense material its speed decreases. The speed of light in a material is generally a function of the wavelength. The *index of refraction* of a material is:

$$\eta(\lambda) = \frac{c}{v_\lambda} \tag{3}$$

where

v_λ**is** the velocity of light in the material

c is the speed of light in a vacuum

The equation that determines the relationship between the angle of incidence, θ_i, and the angle of transmission, θ_r, is *Snell's* law:

$$\eta_i \sin\theta_i = \eta_t \sin\theta_t \tag{4}$$

Here η_i is the index of refraction of the incidence material and η_t is the index of refraction of the transmission material. If the transmission material is denser than the incidence material ($\eta_t > \eta_r$) we can solve the Eq. 4 for all values of $\theta_i = [-\pi/2..\pi/2]$.

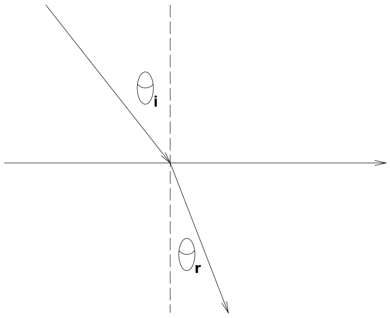

Figure 3: Refraction of light

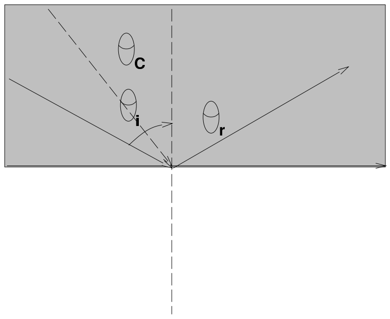

Figure 4: Total internal reflection

If the η_i is larger that η_r then the equation 4 can't be solved for all $\theta_i = [-\pi/2..\pi/2]$. In order to find the range of θ_i that have solutions we can solve the equation for $\theta_r = \pm\pi/2$.

$$\sin\theta_c = \frac{\eta_t}{\eta_i} \tag{5}$$

Here This will give us the angle at which the refracted ray just skims the surface of the interface. For larger angles Eq. 4 has no solutions. This θ_i is called θ_c or the critical angle. Larger angles permit no transmitted light, see Fig. 4, all light is reflected. This is sometimes called *total internal reflection*. This effect is very critical for optical prisms and fiber optics cables. This effect must be detected when creating images of transparent objects.

1.3 Reflectivity functions

There is an obvious equation that reflects the law of conservation of energy:

$$I_i = I_r + I_q + I_b + I_t \tag{6}$$

Where I_i is the incident light, I_r is the reflected light, I_q is the scattered light, I_b is the absorbed light and I_t is the transmitted light. The function that determines the reflectance of light from a point on the surface of the material to a point of view is a complex function of the wavelength and of the three unit vectors $\mathbf{N}, \mathbf{L}, \mathbf{V}$. It can be parameterized by the wavelength, incident angles and viewing angles (Fig 5):

$$R_{bd}(\lambda, \theta_i, \psi_i, \theta_v, \psi_v) = \frac{I_v(\theta_i, \psi_i, \theta_v, \psi_v)}{E_i(\theta_i, \psi_i)} \tag{7}$$

where I_v is the outgoing intensity and E_i is the incoming energy. The energy of the light falling on an area of a surface from a given solid angle $d\omega_i$ is dependent on the cosine of the incident versus the normal to the surface.

$$E_i(\theta_i, \psi_i) = I_i(\theta_i, \psi_i)(N \cdot L)d\omega_i \tag{8}$$

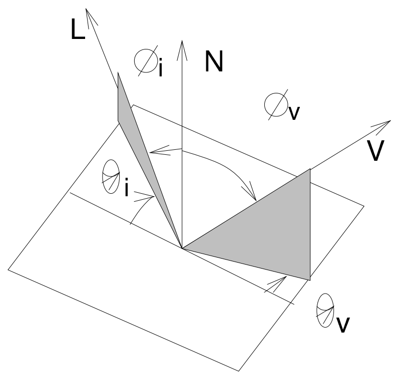

Figure 5: The bi-directional reflectance function

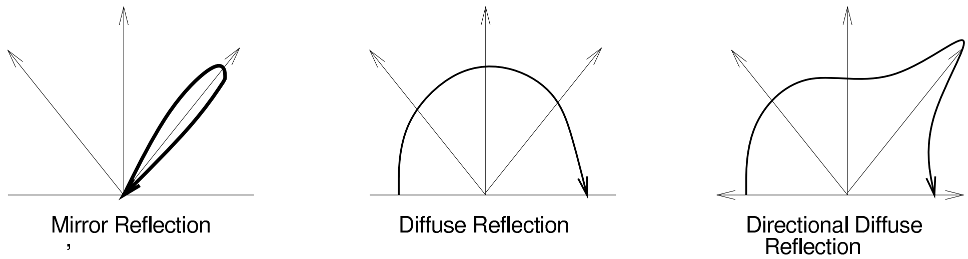

Figure 6: Some cases of interest for reflection

Fig. 6 shows the function R_bd for three cases of interest. The first is the case of a mirror like reflection. The second case is nondirectional diffuse reflection. This might be the reflection of light off a woven material or a matte surface. The third case is a mixture of directional and diffuse reflection. This might correspond to reflection off a shiny plastic surface.

1.4 The Phong reflection model

The *Phong reflection model* imitates the above behavior to a degree that produces acceptable results for simple photo-realistic modeling. It uses a very simple model of the function R_{bd} that has no dependence on the incoming light direction. Nevertheless this model is widely used because of its simplicity.

The model decomposes the reflectance into three components that are treated differently.

$$I_r = I_g + I_d + I_s \tag{9}$$

The three terms are respectively, ambient, diffuse and specular. The ambient term, I_g is simply dependent on the incident ambient illumination and reflecting material, $I_a k_a$ there are no geometric considerations. The diffuse term, I_d is dependent on the direct illumination, the reflecting material and the difference between normal angle of the surface and the illumination direction, $I_i k_d(\mathbf{L} \cdot \mathbf{N})$. The specular term is approximated by a term that is dependent on the difference between the specular reflectance vector and the viewing vector, $I_i k_s(\mathbf{R} \cdot \mathbf{V})^n$. Where n is an empirical parameter that determines the "sharpness" of the specular highlights. The larger n the closer the specular term approaches an ideal (mirror) reflecting surface. The Phong reflectance formula is thus:

$$I = I_a K_a + I_i(K_d(\mathbf{L} \cdot \mathbf{N}) + k_s(\mathbf{R} \cdot \mathbf{V})^n) \tag{10}$$

In general the ambient and diffuse terms have material dependent color reponse. The specular term is less color dependent. The color frequency dependent Phong reflectance formula is thus:

$$I(\lambda) = I_a(\lambda)K_a(\lambda) + I_i(\lambda)(K_d(\lambda)(\mathbf{L} \cdot \mathbf{N}) + K_s(\mathbf{R} \cdot \mathbf{V})^n) \tag{11}$$

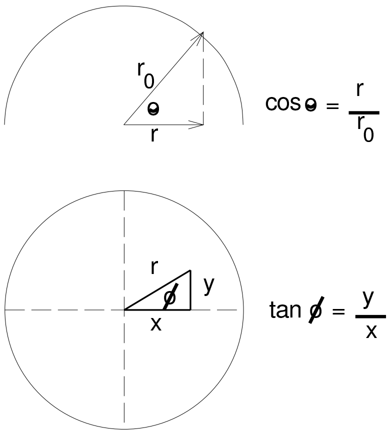

Figure 7: Finding the normal vectors of a spherical surface.

2 Exercises

1. If we want to render an object we have to be able to evaluate the normal vector at its surface. The geometry of a sphere is quite simple and it can be found in Fig. 7. We can think of the upper figure as a cross section of the sphere sliced in the $x - z$ plane. The radius of the sphere is r_0, and the center of the sphere is located at the origin. We can assume for the moment that y value is zero ($\psi = 0$) and $r = x$. We then can see that the normal vector is in the $x - z$ plane. The normal vector at the point $[x, 0, \sqrt{r_0^2 - x^2}]$ is:

$$\mathbf{N}_0 = [\cos\theta, 0, \sin\theta] = [\frac{r}{r_0}, 0, \sqrt{1 - \frac{r^2}{r_0^2}}] \tag{12}$$

   Find the formula for the normal vector at any point on the sphere's surface.